Best Practices

Determining the Discount for Lack of Marketability with Put Option Pricing Models in View of the Section 2704 Proposed Regulations

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Proposed regulations under Internal Revenue Code Section 2704 introduce the use of a six-month put option to estimate the discount for lack of marketability of business ownership interests for gift, estate, and generation-skipping transfer tax transactions. This discussion outlines the various put option models often relied on by valuation analysts to estimate the discount for lack of marketability.

Introduction

Business valuations prepared for gift and estate tax purposes usually involve the valuation of a privately held company, which are nonmarketable (or, at best, privately marketable).

In these valuations, the valuation analyst (analyst) typically estimates the value of the company first as if the underlying shares were publicly marketable. Then, the analyst incorporates a discount for lack of marketability (DLOM) to reflect the fact that the underlying shares are nonmarketable (or privately marketable).

The Internal Revenue Service (the "Service") has issued long-anticipated proposed regulations under Internal Revenue Code Section 2704 that attempt to substantially reduce the application of valuation discounts to intrafamily transfers of interest in entities (such as corporations, partnerships, or limited liability companies).[1]

If initial interpretations prove correct, these regulations may restrict the DLOM to the value of a six-month put option.

This restriction stems largely from the creation of a new category of "disregarded restrictions." Some commentators have viewed this new category of restrictions as effectively valuing transfers of interests in family-controlled entities as if the holder of the interest has a put right to sell the interest to the entity within six months for a value at least equal to a pro rata part of the net value of the entity in return for cash or property.

This discussion introduces and compares put option pricing models to estimate the DLOM for gift and estate tax purposes.

Brief Background of Section 2704

Section 2704 was enacted in 1990 with the goal of limiting discounts for certain family-owned, or closely held, interests that are transferred to family members.[2]

If an individual and the individual's family hold voting or liquidation control over a corporation or partnership, Section 2704(a) provides, in general, that the lapse of a voting or liquidation right shall be taxed as a transfer subject to gift or estate tax.

If an "applicable restriction" limits the ability of the corporation or partnership to liquidate, and that restriction can be removed by the family, then Section 2704(b) provides that the restriction is

disregarded in valuing the transferred interest for gift or estate tax purposes.

Under the existing regulations, an applicable restriction does not include "any restriction imposed, or required to be imposed, by any Federal or State law" (or commercially reasonable restrictions imposed by unrelated persons in a financing transaction).

This provision has been interpreted by the Service to mean that default restrictions on the ability of an owner to withdraw from a subject entity could be considered, even though the family could have overridden those restrictions in the governing documents.

Many states have a default rule limiting the ability of a limited partner or member of a limited liability company to withdraw, and the Service stated that the default rule in the regulations has made Section 2704(b) "substantially ineffective."[3]

The proposed regulations eliminate this default rule by providing that applicable restrictions must be mandated by federal or state law (and thus not permitted to be overridden by the family) in order to be considered in determining the fair market value of the transferred interest.

The proposed regulations further limit the valuation discounts with the introduction of the "disregarded restrictions."

This new category of restrictions is defined as a provision of the governing documents or applicable law that limits the ability of the interest holder to compel liquidation or redemption of an interest on no more than six months' notice for cash or property equal at least to what the proposed regulations call "minimum value."[4]

The regulations do not include specific examples of corporations or limited partnerships or limited liability companies that are merely silent on the ability of a shareholder, limited partner, or member to withdraw and have the interest redeemed by the subject entity.

Commentators generally have read into the proposed regulations a deemed put right. They note that the disregarded restrictions provision may have very little impact on valuation if it is not interpreted to value transferred interests as if a six-month put right at minimum value exists.[5]

Overview of the Put Option Pricing Models

A put option, simply stated, is an option to sell financial assets at an agreed price on or before a particular date. Put options are based on financial option pricing theory. This theory has been used to explain the purportedly "irrational" pricing that is observed in certain situations in the capital markets, as conventional methods may understate the intrinsic value of a financial asset.

Put option pricing models (POPMs) have been applied to estimate the DLOM for private company interests. Despite some shortcomings, POPMs are still one of the few available techniques to actually quantify a DLOM.

"[L]ack of liquidity is a form of DLOM that exists when an interest holder cannot dispose of the interest quickly unless the holder is willing to accept a significant reduction in value."

Finnerty stated that a lack of liquidity is a form of DLOM that exists when an interest holder cannot dispose of the interest quickly unless the holder is willing to accept a significant reduction in value.[6]

He concluded that this lack of liquidity, and by extension the DLOM, can be estimated based on a POPM:

> One can also model the cost of the lack of liquidity as the value of a forgone put option. However, the option formulation is more complex than in the case of the lack of marketability because there is no legal or contractual restriction on the holder's ability to sell or transfer the asset, and, consequently, the length of the restriction period is less clear. For example, the market for an asset may be poorly developed, making it difficult, time-consuming, and therefore expensive to find a buyer for the securities, but the assets are nevertheless marketable. The restrictions are financial, rather than legal or contractual, and there is no fixed date on which they are scheduled to lapse. It takes more time to find a buyer in an illiquid market than in a liquid market. This loss of flexibility to sell an asset freely or, equivalently, the ability to sell it quickly but only if there is some concession of intrinsic value, can be modeled as the loss of value of a put option.[7]

Chaffe, an early pioneer in applying POPMs to estimate the DLOM, wrote that by purchasing a put option, a restricted stock (i.e., one that is exercisable only at the end of the option period) would reasonably replicate the lapsing of Securities and Exchange Commission (SEC) Rule 144 restrictions.[8,9]

It is instructive, therefore, to view the results of the POPMs through the lens of restricted stock studies. Exhibits 1a and 1b present a summary of restricted stock studies categorized into three time periods related to changes in the holding period provision of SEC Rule 144.[10]

These restricted stock studies generally indicate a decrease in the average DLOM after 1990. The restricted stocks analyzed in the studies covering the 1968 to 1988 period (where the average indicated DLOM was approximately 35 percent) were generally less marketable than the restricted stocks analyzed after 1990 (where the average indicated DLOM was typically less than 25 percent).

Analysts typically attribute this decrease in price discounts to the following reasons:

1. There was an increase in volume of privately placed stock under SEC Rule 144(a).
2. The minimum SEC-required holding period under Rule 144 was reduced—from two years to one year—as of April 29, 1997.[11]

Increased volume was the result of a Rule 144 amendment in 1990 that allowed qualified institutional investors to trade unregistered securities among themselves. By increasing the potential buyers of restricted securities, the marketability of these securities generally increased.

As it became easier to find a buyer for restricted securities after 1990, the average restricted stock price discount decreased. The same trend occurred after the SEC-required holding period decreased from two years to one year in 1997.

On December 17, 2007, the SEC issued revisions to Rules 144 and 145.[12]

The revisions included shortening the holding period for restricted securities of issuers that are subject to the Securities Exchange Act of 1934 reporting requirements from one year to six months for an issuer that has been a reporting company for at least 90 days. This amendment became effective February 15, 2008.

Valuation analysts typically compare the market for the subject privately held company with the market for restricted securities in view of the expected holding period. If the subject privately held stock is likely to be liquidated within six months, as is the case under the Section 2704 proposed regulations, the post-1997 studies may be the most meaningful (reflecting a holding period of six months to one year).

While the post-2007 studies reflect a more limited holding period of six months, Stockdale points out that there is no statistical difference in the average DLOM between the SEC Rule 144 one-year and 0.5-year holding periods based on a statistical study of the FMV Opinions Restricted Stock Study through 2011 (the "FMV study") and Pluris Valuation Advisors DLOM databases.[13]

The FMV study is particularly instructive because it incorporated data from 1980 through 2015, and information is available about each of the 736 restricted stock transactions included in the study.[14]

The companion guide to the FMV study classifies the results into quintiles based on several financial metrics, including volatility, as presented in Exhibit 2.[15] This classification is helpful in evaluating the relevance and application of the available POPMs.

A logarithmic regression analysis of the quintile results suggests that volatility in the range of approximately 65 percent to 105 percent is strongly correlated to the median quintile discounts. This range corresponds to the typical volatility observed for privately held companies. This relationship does not appear to hold for lower volatilities.

The curve fitting of these data is presented in Figure 1.

In conjunction with the FMV study and the other restricted stock studies summarized above, five commonly known POPMs have emerged over the years for estimating the DLOM:

1. The Chaffe model
2. The shout put option model
3. The Longstaff model
4. The Finnerty model
5. The Ghaidarov model

These five POPMs are summarized in Exhibit 3. Their respective application to estate and gift tax matters is discussed in the sections that follow.

Chaffe Put Option Model

Chaffe introduced the concept of using a POPM to estimate the DLOM.[16] Because a DLOM results from an inability to exercise a right to sell, the cost of the put reflects the DLOM for the shares. The put option value divided by the stock price represents the percentage DLOM.

Like the Black-Scholes model (BSM), the Chaffe model is based on several assumptions characteristic of option models, including the following:

- The option trades on an organized and liquid exchange, providing for efficient trading.

Exhibit 1a
Restricted Stock Study

	Period Covered		Number of	Reported	Reported
	From	To	Observations	Median	Mean
SEC Overall Average	1966	1969	398	24%	26%
Johnson and Racette	1967	1973	86	NA	34%
Milton Gelman	1968	1970	89	33%	33%
Robert R. Trout	1968	1972	60	NA	34%
Robert E. Moroney	1969	1972	146	34%	35%
J. Michael Maher	1969	1973	34	33%	35%
Stryker/Pittock	1978	1982	28	45%	NA
Wruck, Karen H.:					
Registered	1979	1984	36	2%	-4%
Unregistered	1979	1984	37	12%	14%
FMV Opinions (Hall/Polacek)	1979	1992	100+	NA	23%
Barclay, Holderness, and Sheehan	1979	1997	594	17%	19%
Hertzel and Smith	1980	1987	106	13%	20%
Management Planning, Inc.	1980	1995	49	29%	28%
Management Planning, Inc.	1980	1995	20	29%	27%
Hertzel, Lemmon, Linck, and Rees	1980	1996	404	13%	17%
Willamette Management Associates	1981	1984	33	31%	NA
Silber (1981-1988)	1981	1988	69	NA	34%
Krishnamurthy, Spindt, Subramanium, and Woidtke:					
All	1983	1992	391	NA	19%
Restricted Shares	1983	1992	75	NA	34%
Shares with Registration Pending	1983	1992	23	NA	23%
Shares Not Known to Be Restricted	1983	1992	293	NA	15%
Shares with Pending Registration or Not Known	1983	1992	316	NA	16%
Wu	1986	1997	301	20%	9%
Bajaj, Denis, Ferris, Sarin:					
All	1990	1995	88	21%	22%
Registered	1990	1995	37	10%	14%
Unregistered	1990	1995	51	27%	28%
FMV Opinions (1991–1992)	1991	1992	NA	NA	21%
BVR (Johnson)	1991	1995	72	NA	20%
Columbia Financial Advisors	1996-April	1997	23	NA	21%
All Studies through 1997				**23%**	**23%**

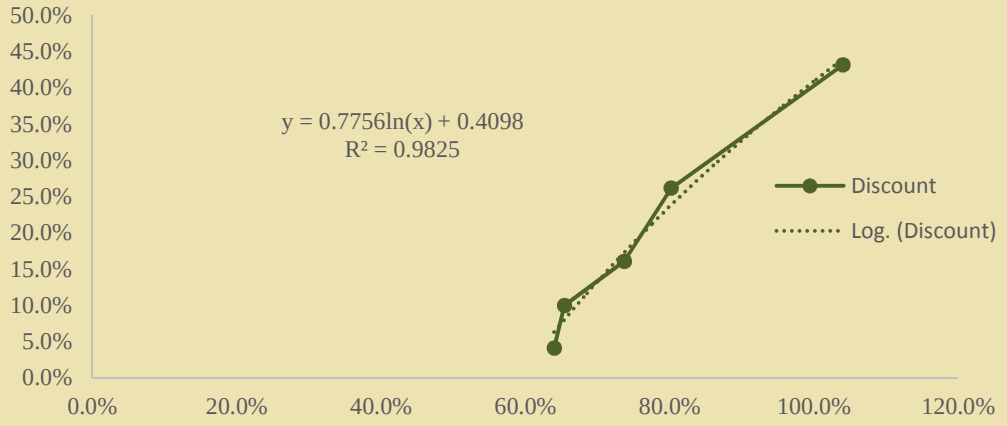
Exhibit 1b
Restricted Stock Study

	Period Covered		Number of	Reported	Reported
	From	To	Observations	Median	Mean
Columbia Financial Advisors	1996	1997	15	9%	13%
Verdasca	2000	2006	771	10%	10%
Glegg, Harris, Madura, and Ngo	2000	2008	601	8%	9%
Billett and Floros	2001	2008	12,004	27%	NA
All Studies after 1997 and before 2008				**14%**	**11%**
Wruck and Wu	1980	1999	1,854	11%	11%
Angrist, Curtis, and Kerrigan (MPI)	1980	2009	1,863	13%	16%
Finnerty:					
Pre-February 1997	1991	2007	41	20%	26%
Post-February 1997	1991	2007	176	16%	22%
FMV Opinions (1997–2007)	1997	2007	NA	16%	20%
FMV Opinions (2001–2007)	2001	2007	NA	13%	15%
Chaplinsky and Haushalter:					
Purchase Discount Only	1995	2000	382	17%	19%
Purchase Discount and Warrant	1995	2000	235	14%	17%
Brophy, Ouimet, and Sialm:					
Hedge Funds - Traditional PIPEs	1995	2002	586	NA	14%
Other Investors - Traditional PIPEs	1995	2002	1,559	NA	9%
Floros and Sapp	1995	2008	14,391	11%	NA
Huson, Malatesta, and Parrino	1995	2009	1,029	12%	12%
Meidan	1996	2006	1,726	NA	10%
All Studies Spanning 1997				**14%**	**16%**
Stout Risius Ross	2005	2010	98	9%	11%
FMV Opinions (2008–2011)	2008	2011	NA	10%	12%
Harris-Trugman Valuation Associates					
All	2007	2010	136	14%	17%
Pre-SEC Rule Change	2007	2010	47	15%	18%
Post-SEC Rule Change	2007	2010	89	14%	16%
All Studies Spanning 2008				**15%**	**13%**

Exhibit 2
FMV Opinions Restricted Stocks Study (1980–2015)
Results by Quintile

	Quintile				
	1	2	3	4	5
Number of Tranactions	147	147	148	147	147
Discount:					
Low	0.0%	7.4%	13.0%	20.9%	33.9%
High	7.4%	13.0%	20.8%	33.5%	91.3%
Median	4.1%	10.0%	16.1%	26.2%	43.2%
Company Characteristics:					
Volatility	64.0%	65.4%	73.7%	80.2%	104.0%
Market Value ($MM)	183.5	194.2	116.6	102.1	57.7
Market-to-Book Ratio	2.6	3.2	3.6	5.7	6.1
Total Assets ($MM)	112.9	88.1	37.0	23.1	11.4
Revenue ($MM)	31.7	42.3	20.9	17.1	8.4
Net Profit Margin	-6.7%	-5.4%	-8.3%	-23.4%	-38.7%

Figure 1
Curve Fitting to FMV Study Quintile Data
Discount as a Function of Volatility



Volatility	65.0%	75.0%	85.0%	95.0%	100.0%	115.0%	125.0%
Implied DLOM	7.6%	18.7%	28.4%	37.0%	41.0%	51.8%	58.3%

Exhibit 3
Summary of Put Option Pricing Models

POPM	Characteristics	Typical DLOM for Six-Month Holding Period [a]	Strength(s)	Weakness(es)
Chaffe	European fixed strike	14.0% to 33.5%	Based on well-known Black-Scholes-Merton (BSM) option theory; may be more representative of restricted stock studies at lower volatilities than other POPMs	Ignores risk of price increases; indicated DLOM is excessively high at higher volatilities
Shout Put	European fixed strike plus a shout premium	14.0% to 33.5% (same as Chaffe for short holding periods)	Lock in a minimum payout while retaining the right to gain from price increases	Ignores risk of price increases; indicated DLOM is excessively high at higher volatilities; more complicated model
Longstaff (transformed)	American fixed strike lookback	23.9% to 48.0%	Considers risk of price increases during holding period	Assumes perfect market timing ability; may overstate the DLOM at higher volatilities
Finnerty (modified)	Asian-style average strike	8.0% to 18.8%	Considers risk of price increases during holding period without perfect market timing ability	Indicated DLOM is limited to 32.3% regardless of higher volatilities or longer time; tends to produce a minimum DLOM
Ghairadov	Asian-style average strike	8.2% to 25.2%	Considers risk of price increases during holding period without perfect market timing ability	Tends to produce a minimum DLOM

[a] Based on a six-month holding period, a risk-free interest rate of 2.0 percent, and share price volatility in the range of 50 percent to 125 percent.

- Standardized terms exist for the option contracts.
- Both the interest rate and volatility of the stock price are constant throughout the holding period.
- Stock prices are continuous, which ignores the potential for sudden extreme jumps, such as those often experienced in the aftermath of an announcement of an acquisition or merger.
- The holder possesses some degree of market timing ability (i.e., the holder would sell only if the holder knows the share price will fall below the strike price at expiration).

These assumptions often run counter to economic reality. Holders of restricted stock trade on an organized and liquid exchange and understand that the public company stock will be tradable on the exchange following the restriction period.

The holder of a privately held company equity interest has no access to a comparable exchange and no assurance of a sale. A put option value, therefore, reflects the additional risk of holding privately held company equity interests.

Chaffe relied on the BSM option pricing model to estimate the price of the option in his model. The inputs to the Chaffe model are the same as the BSM model and reflect the adaptation of BSM financial option value theory, which was derived from studies of publicly traded stocks, to privately held company interests.

$$DLOM = \frac{Xe^{-rT} \times N(-d_2) - Se^{-qT} \times N(-d_1) - S + Xe^{-rT}}{S}$$

where

$$d_1 = \frac{\ln(\frac{S}{X}) + T(r - q + \frac{\sigma^2}{2})}{\sigma\sqrt{T}} \text{ and } d_2 = d_1 - \sigma\sqrt{T}$$

The inputs in the Chaffe model and the effect on DLOM are as follows:

Variable	Variable Description	Effect of Change on DLOM
S	Current price of the security	No effect where S=X
X	Exercise (strike) price	No effect where S=X
N(·)	Cumulative probability function for a standardized normal distribution	No effect
ln(·)	Natural logarithm function	No effect
e	The base of the natural logarithm	No effect
T	Time to expiration (in years)	Increases as time increases
r	Risk-free interest rate	Increases as interest rate increases
q	Dividend yield (if any)	Decreases as yield increases
σ	Annualized volatility of the security price	Increases as volatility increases

In particular, the stock price and strike price equal the marketable value of the privately held company stock as of the valuation date; the time to expiration equals the time the securities are expected to remain nonmarketable after the sell decision has been made (i.e., the holding period); the interest rate is the risk-free cost of capital; and, the volatility, a judgmental factor, is often estimated by reference to the volatility of guideline publicly traded stocks (i.e., the standard deviation of annual returns).

Volatility may be estimated based on a number of approaches, including a review of guideline publicly traded company stock price volatility over various time periods.

Selecting the holding period for a privately held company interest is a matter of professional judgment, but it is seldom less than a few months. The holding period encompasses the period necessary to complete marketing and selling activities, including the following:

- Developing a marketing strategy
- Drafting marketing and selling documents
- Implementing the marketing strategy
- Identifying and screening buyers
- Assisting buyers in their analysis of the company and the interest being sold
- Conducting site visits for buyers
- Drafting letters of intent
- Negotiating with qualified buyers
- Assisting buyers with due diligence
- Drafting the contract of sale
- Participating in arranging financing
- Closing the sale

The relationship between volatility and holding time of stock sales is illustrated in Exhibit 4.

In view of the proposed Section 2704 regulations, the Service appears to assume a maximum holding period of six months for intrafamily transactions. The Service's rationale for the shortened period apparently is based on the assumption that the family could purchase the holder's interest upon request (the deemed put right), thereby eliminating some, if not most, of the marketing and selling activities.

For illustrative purposes, we will assume a risk-free interest rate of 2.00 percent and no dividends. Exhibit 5 presents representative DLOMs based on the Chaffe model with these inputs. According to the Chaffe model, the implied DLOM is between 6.5 percent and 70.3 percent for stocks with volatility between 25.0 percent and 3.0 percent and a holding period of six months.

Chaffe noted that his findings were downward biased due to the reliance on a European put option in his model. For a European put option, the holder is presumed to hold the ownership interest in the privately held company until the end of the holding period, which decreases the option value relative to the American option, which can be exercised earlier. Therefore, Chaffe concluded that his findings may be viewed as a minimum applicable discount.

Chaffe's end-of-period assumption also underscores one of the criticisms of protective puts being a reasonable proxy for the DLOM: it mitigates downside risk but does not address upside risk. Some commentators have suggested that the discount indicated by the put option should be offset by the value of a written call option with terms matching the put option.

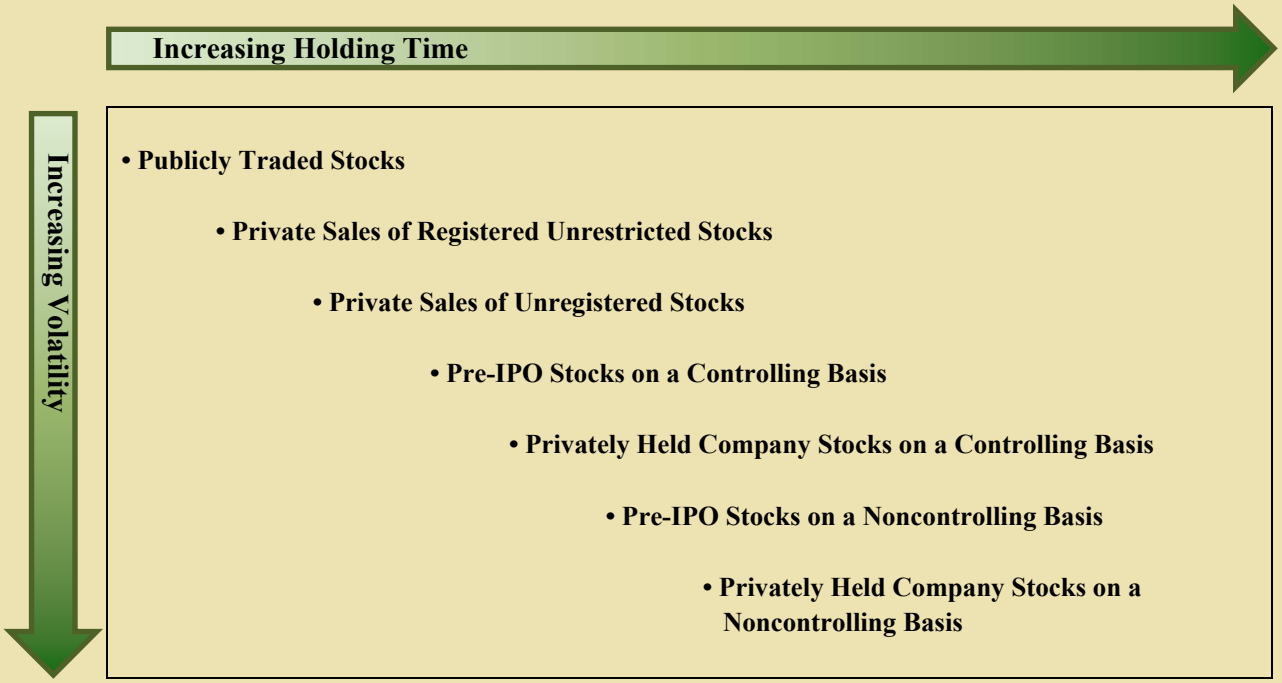
According to Chaffe, volatility for a small privately owned company is likely to exceed 50 percent. Chaffe reached this conclusion based on the volatility for small public companies that are traded in the over-the-counter market.

The Chaffe study found that the indicated DLOM for a privately held stock with a six-month required holding period and volatility between 50 percent and 125 percent is between approximately 14 percent and 34 percent.

Although this is a large range for these DLOMs, the results generally are consistent with other DLOM studies such as restricted stock studies and pre-initial-public-offering studies.

In order to analyze the reasonableness of the Chaffe model output, this discussion compares the

Exhibit 4
Stock Sales



implied DLOM under various scenarios to the results of the FMV study presented in Exhibit 2.

According to the Chaffe model (Exhibit 5), under a one-year holding period and assuming 75 percent volatility, the implied DLOM is 28 percent. This DLOM is similar to the fourth quintile median DLOM of 26.2 percent in the FMV study, which reports 80.0 percent volatility (Exhibit 2).

Exhibit 5
Indicated DLOM Based on the Chaffe Put Option Model

Holding Period (T, Years)	Volatility (σ)								
	25%	50%	75%	100%	125%	150%	200%	250%	300%
0.25	4.7%	9.7%	14.6%	19.4%	24.2%	28.9%	37.9%	46.4%	54.3%
0.50	**6.5%**	**13.5%**	**20.3%**	**27.0%**	**33.5%**	**39.7%**	**51.3%**	**61.5%**	**70.3%**
0.75	7.8%	16.3%	24.5%	32.5%	40.1%	47.3%	60.2%	70.8%	79.3%
1.00	8.9%	18.6%	28.0%	36.9%	45.4%	53.1%	66.6%	77.1%	84.8%
2.00	11.8%	25.2%	37.7%	49.1%	59.2%	67.8%	80.7%	88.5%	92.8%
3.00	13.8%	29.6%	44.1%	56.7%	67.1%	75.4%	86.1%	91.2%	93.3%
4.00	15.2%	33.0%	48.8%	61.8%	72.0%	79.5%	87.9%	91.1%	92.1%

Assumptions: S = X, r = 2.00%, q = 0%

Stockdale suggests that the Chaffe model is best used at low volatilities (below 50 percent) and the holding period is relatively short because the model tends to generate discounts that exceed the discounts indicated for the observed transaction data.[17]

This deviation is evident for long holding periods. However, for a six-month holding period, it appears that the Chaffe model may be useful at the higher volatilities ordinarily associated with privately held companies.

A comparison of the Chaffe model results to the FMV study discounts summarized in Exhibit 2 and Figure 1 suggests that the Chaffe model results are comparable and may even understate the DLOM, as presented in Exhibit 6.

Indeed, further upward adjustment in the put option value may be warranted to account for information asymmetry and the difficulty in applying the volatility observed for guideline publicly traded stocks to the privately held company.

In *Estimating the Cost of Capital—Applications and Examples*, Pratt and Grabowski explain that certain put option models, like the Chaffe model, tend to understate the DLOM:

> The option models are all based on an underlying publicly traded stock. The option model results lack the reality of what an investor would require in terms of fair return, namely an option on a nonmarketable security. Empirical data suggests that institutions active in the "market" for private warrants purchase them at a significant discount to their theoretical Black-Scholes formula value because of their illiquidity, implying greater discounts.
>
> As a result, discounts drawn from restricted stock transactions and implied by put option models underestimate the discount for lack of marketability for stock in a closely held business.[18]

Shout Put Option Model

Katsanis advocated the application of a shout put option model for estimating the DLOM.[19]

According to Katsanis, the shout put option value serves as an estimate of the marketability and liquidity value embedded within the marketable share value so that the following relationship exists:

Marketable Share Value =
Shout Put Value + Nonmarketable Share Value

The shout put option model is essentially a modification of the Chaffe model. If the risk-free interest rate exceeds the dividend yield and the dividend yield is not zero, the put value concluded by the Chaffe model is multiplied by an exponential adjustment factor based on the expected dividend yield of the subject company security.

Exhibit 6
Relationship of DLOM Results and Volatility

Volatility	Chaffe Model Implied DLOM	FMV Study Implied DLOM
75.0%	20.3%	18.7%
100.0%	27.0%	41.0%
125.0%	33.5%	58.3%

The equation for the shout put option model is as follows:

$$Put\ value = \begin{cases} CM(\cdot), & if\ r \le q \\ CM(\cdot), & if\ r > q, \tau \in (0, \tau^*] \\ e^{-q(\tau-\tau^*)}CM(\cdot), & if\ r > q, q > 0, \tau \in (\tau^*, \infty) \end{cases}$$

where:

CM(·) = The Chaffe model equation

Katsanis explains:

> European style fixed strike, lookback, and Asian put option models have been proposed and utilized by business valuation practitioners to estimate discounts for lack of marketability. Another form of put option, a shout put or shout floor option, more closely mimics marketability than do the previously mentioned forms of put option because both marketability and a shout put option give a stockholder the right to lock in a selling price (the prevailing marketable stock price) for the stock at any point in time over the term of the option. By comparison, over the term of the option the European fixed strike put gives the stockholder the right to lock in a selling price equal to the current stock price; the lookback put gives the stockholder the right to lock in a selling price equal to the highest stock price achieved; and the Asian put gives the stockholder the right to lock in a selling price equal to the average of all stock prices achieved. Because the shout put option more closely mimics marketability than do the alternative put option models, it would be a valuable addition to every valuation practitioner's toolbox.[20]

In his book *Discount for Lack of Marketability*,[21] Gregory, a valuation analyst and former Service agent who led the initial development of the Service's DLOM Job Aid, recommends consideration of the shout put model for holding periods longer than one year, as it may provide a somewhat more accurate estimate of the DLOM than the Chaffe model.

However, for shorter holding periods, such as six months under the proposed Section 2704 regulations, any improved accuracy provided by the shout put option model likely is insignificant and not worth the added complexity.

The DLOM incorporating a 10 percent dividend yield and a one-month differential time period (τ-τ*), for example, equals:

$$Unadjusted\ DLOM\ (per\ Chaffe\ model) = 20.00\%$$
$$Adjusted\ DLOM = e^{-q(\tau-\tau^*)} \times 20.00\% = e^{-0.1(0.822)} \times 20.00\% = 19.84\%$$
$$Change\ in\ DLOM = \frac{20.00\%}{19.84} - 1 = -0.82\%$$

The example adjustment represents less than a 1 percent change in the DLOM, which is not statistically significant under ordinary circumstances.

Longstaff Lookback Put Option Model

Longstaff authored a study that relied on stock options to estimate the upper bound of a DLOM for a privately held company.[22]

Whereas Chaffe based his model on avoiding losses, Longstaff based his model on unrealized gains. Another difference is that the Longstaff model purportedly provides an estimate for the upper limit on the value for marketability. The Longstaff model is based on the price of a hypothetical "lookback" option, which is a type of American option that permits the option to be exercised prior to the expiration date.

A "lookback" put option differs from most other put options in that the holder can look back at the end of the put option's life and retroactively exercise the option at the highest stock price during the holding period, yielding the maximum return.

The Longstaff model assumes an investor has a single-security portfolio, perfect market timing, and trading restrictions that prevent the security from being sold at the optimal time. The value of marketability, based on these assumptions, is the payoff from an option on the maximum value of the security, where the strike price of the option is stochastic.

Longstaff explains:

> [Consider] a hypothetical investor with perfect market timing ability who is restricted from selling a security for T periods. If the marketability restriction were to be relaxed, the investor could then sell when the price of the security reached its maximum. Thus, if the marketability restriction were relaxed, the incremental cash flow to the investor would essentially be the same as if he swapped the time-T value of the security for the maximum price attained by the security. The present value of this lookback or liquidity swap represents the value of marketability for this hypothetical investor, and provides an upper bound for any actual investor with imperfect market timing ability.[23]

The equation for the Longstaff lookback put option model is as follows:

$$Put\ value = V_0\left[(2+\frac{\sigma^2 T}{2}) \times N(\sqrt{\frac{\sigma^2 t}{2}}) + \sqrt{\frac{\sigma^2 T}{2\pi}} e^{-\frac{\sigma^2 T}{8}} - 1\right]$$

where:

- $N(\cdot)$ = Cumulative probability function for a normal distribution
- T = Time to expiration of put option (in years); holding period
- e = A mathematical constant—the base of the natural logarithm
- Π = A mathematical constant
- σ = Annualized volatility of the underlying security
- V_0 = Value of the otherwise identical unrestricted interest

Exhibit 7 presents DLOMs based on holding periods from one-quarter of a year to four years and volatility inputs from 25 percent to 300 percent. There is disagreement among valuation analysts whether the results produced by the Longstaff model reflect a liquidity premium or a DLOM.

When comparing the original Longstaff model results (Exhibit 7, wherein results are assumed to reflect a discount) to the FMV Opinions Study results (Exhibit 2), the Longstaff model reports DLOMs that (1) are far greater than the observed discounts from restricted stock transactions and (2) exceed 100 percent at reasonable levels of volatility—an illogical conclusion.

Abbot believes that the Longstaff model results in a premium:

> Often, however, the value of a put option premium, estimating the cost of liquidity, is presented incorrectly as the discount for lack of liquidity. This is similar to the merger premium being treated as a discount for lack of control. Neglecting to convert the option premium to the applicable discount creates the illusion that the estimated discounts are greater than 100%, an impossible solution.[24]

Abbot advocates the following transformation of the Longstaff model in order to convert the model result (assumed to be a liquidity premium) into a DLOM:[25]

$$DLOM = \frac{put\ value}{1+put\ value}$$

Exhibit 8 presents DLOMs per the transformed Longstaff model based on holding periods from one quarter of a year to 4 years and volatility inputs from 25 percent to 300 percent. While still representing an upper bound, the transformed Longstaff model results conform more closely to the FMV study than the results of the original Longstaff model. Also, the results do not violate the 100 percent theoretical maximum.

In its DLOM Job Aid, the Service states that the Longstaff model is not often applied by valuation analysts in estimating the DLOM for a privately held company.[26]

The Longstaff model assumes that an investor has perfect timing, which defies economic reality. Longstaff also analyzed securities with a volatility

Exhibit 7
DLOMs for Original Longstaff Put Option Model
Impact of Holding Period and Volatility on DLOM Six-Month Holding Period Highlighted

Holding Period (T, Years)	Volatility (σ)								
	20%	50%	75%	100%	125%	150%	200%	250%	300%
0.25	8.2%	21.6%	33.6%	46.6%	60.4%	75.3%	108.1%	145.2%	186.9%
0.50	**11.8%**	**31.5%**	**49.8%**	**70.1%**	**92.3%**	**116.7%**	**172.0%**	**236.9%**	**311.9%**
0.75	14.6%	39.5%	63.3%	90.0%	119.8%	153.0%	229.9%	321.9%	430.0%
1.00	17.0%	46.6%	75.3%	108.1%	145.2%	186.9%	284.9%	404.0%	545.4%
2.00	24.6%	70.1%	116.7%	172.0%	236.9%	311.9%	494.3%	722.8%	999.2%
3.00	30.8%	90.0%	153.0%	229.9%	321.9%	430.0%	697.5%	1036.8%	1449.8%
4.00	36.1%	108.1%	186.9%	284.9%	404.0%	545.4%	898.9%	1349.8%	1900.0%

Assumptions: S = X, r = 2.0%, q = 0%

Exhibit 8
DLOMs for Transformed Longstaff Put Option Model
Impact of Holding Period and Volatility on DLOM Six-Month Holding Period Highlighted

Holding Period (T, Years)	Volatility (σ)								
	20%	50%	75%	100%	125%	150%	200%	250%	300%
0.25	7.6%	17.7%	25.2%	31.8%	37.7%	43.0%	51.9%	59.2%	65.1%
0.50	**10.6%**	**23.9%**	**33.3%**	**41.2%**	**48.0%**	**53.8%**	**63.2%**	**70.3%**	**75.7%**
0.75	12.7%	28.3%	38.8%	47.4%	54.5%	60.5%	69.7%	76.3%	81.1%
1.00	14.5%	31.8%	43.0%	51.9%	59.2%	65.1%	74.0%	80.2%	84.5%
2.00	19.8%	41.2%	53.8%	63.2%	70.3%	75.7%	83.2%	87.8%	90.9%
3.00	23.5%	47.4%	60.5%	69.7%	76.3%	81.1%	87.5%	91.2%	93.5%
4.00	26.5%	51.9%	65.1%	74.0%	80.2%	84.5%	90.0%	93.1%	95.0%

Assumptions: S = X, r = 2.0%, q = 0%

between 10 percent and 30 percent on the premise that this range of volatility is consistent with typical stock return volatilities; however, small stocks (such as those traded over the counter and analyzed by Chaffee) typically have greater volatility (exceeding 50 percent), all else being equal.

To help mitigate the aforementioned deficiencies, Vianello suggests applying an average volatility input instead of a peak volatility.[27]

Vianello calculates a proxy for the subject company's stock price volatility the annualized average stock price volatility for each of the selected guideline companies for a historical period of time equal to the holding period of the subject company security.

Vianello explains:

> It is irrefutable that the cost of illiquidity must be less for the average investor with imperfect market timing than it is for an investor possessing perfect market timing. But the upper bound criticism resulting from this situation is nonetheless defective in the valuation context because it is easily circumvented by using volatility estimates that represent average, not peak, volatility expectations. For example, the appraiser's volatility estimate may be based on some average or regression of historical price volatility derived from an index or from one or more publicly traded guideline companies. Using average volatility estimates in the lookback option formula necessarily results in a value that is less than the upper bound value. Indeed, a value calculated using average expected volatility necessarily suggests a result that is achievable by the average imperfect investor. The resulting value determined in this manner appropriately falls short of a value based on perfect market timing while providing an important informational symmetry lacking in Dr. Longstaff's more simplified framework.[28]

Finnerty Average-Strike Put Option Model

Finnerty conducted an option-pricing study that "tests the relative importance of transfer restrictions on the one hand and information and equity ownership concentration effects on the other in explaining private placement discounts."[29]

The Finnerty option-pricing study is an extension of the Longstaff study. However, unlike Longstaff, Finnerty did not assume that investors have perfect market timing ability. Finnerty modeled the DLOM as the value of an average-strike Asian put option. That is, the Finnerty model assumes that the strike price would be equal to the arithmetic average of market prices over the holding period, rather than the optimal price.

Finnerty's initial model contained a mathematical error that resulted in DLOMs exceeding 100 percent for long holding periods. Finnerty published a modified model that does not violate the 100 percent theoretical limit.[30]

References to the Finnerty model hereafter in this discussion pertain to the modified version unless stated otherwise.

The equation for the Finnerty average-strike put option model is:

$$DLOM = V_0 e^{-qT}\left[N\left(\frac{v\sqrt{T}}{2}\right) - N\left(-\frac{v\sqrt{T}}{2}\right)\right]$$

where:

$$v\sqrt{T} = \sqrt{\sigma^2 T + \ln\left[2\left(e^{T\sigma^2} - \sigma^2 T - 1\right)\right] - 2\ln\left(e^{T\sigma^2} - 1\right)}$$

N(·) = Cumulative probability function for a normal distribution

ln(·) = The natural log function

e = A mathematical constant; the base of the natural logarithm

q = Annualized dividend yield of security

r = Risk-free interest rate

T = Time to expiration of put option (in years)—i.e., holding period

σ = Annualized volatility of the underlying security

V_0 = Value of the otherwise identical unrestricted interest

Many commentators, including Stockdale, consider the Finnerty model to more closely reflect the discounts observed from the FMV study data than the other POPMs at low volatilities.[31]

As presented in Exhibit 9, the Finnerty model generates DLOMs that are relatively close to the average DLOMs reported in the FMV study at volatility in the range of 50 percent to 100 percent. Assuming 75 percent volatility and a one-year holding period, the Finnerty model returns a DLOM of 16.3 percent (Exhibit 9).

The FMV study indicates a DLOM of 18.7 percent at 75.0 percent volatility (Figure 1).

With regard to his option-pricing study, Finnerty concluded that his model "calculates transferability discounts that are consistent with the range of discounts observed empirically in letter-stock private placements for common stocks with volatilities between 30 percent and 70 percent but the implied discounts are greater than/(less than) those predicted by the model for lower/(higher) volatilities."[32]

The Finnerty model, as modified, produces no discount in excess of 32.3 percent regardless of ever higher volatilities and longer holding times. This limitation may significantly understate the FMV study implied DLOM for volatilities exceeding 125 percent and a six-month holding period. This point is further illustrated in the summary discussion below.

Ghaidarov Average-Strike Put Option Model

Ghaidarov developed an average-strike Asian put option model in the course of critiquing the original Finnerty model.[33] As discussed above, Finnerty revised his model as a result of this criticism.

The equation for the Ghaidarov average-strike put option model is as follows:

$$DLOM = V_0 e^{-qT}\left[2N\left(\frac{V_T}{2}\right) - 1\right]$$

where:

$$V_T = \sqrt{\ln\left[2\left(e^{T\sigma^2} - \sigma^{2T} - 1\right)\right] - 2\ln\left(\sigma^2 T\right)}$$

N(·) = Cumulative probability function for a normal distribution

ln(·) = The natural log function

e = a Mathematical constant; the base of the natural logarithm

q = Annualized dividend yield of security

r = Risk-free interest rate

Exhibit 9
Indicated DLOM Based on the Finnerty Put Option Model
Six-Month Holding Period Highlighted

Holding Period (T, Years)	Volatility (σ)								
	20%	50%	75%	100%	125%	150%	200%	250%	300%
0.25	2.3%	5.7%	8.5%	11.2%	13.9%	16.3%	20.9%	24.6%	27.5%
0.50	**3.3%**	**8.0%**	**11.9%**	**15.5%**	**18.8%**	**21.8%**	**26.6%**	**29.7%**	**31.4%**
0.75	4.0%	9.8%	14.4%	18.5%	22.2%	25.3%	29.5%	31.5%	32.1%
1.00	4.6%	11.2%	16.3%	20.9%	24.6%	27.5%	30.9%	32.1%	32.3%
2.00	6.5%	15.5%	21.8%	26.6%	29.7%	31.4%	32.2%	32.3%	32.3%
3.00	7.9%	18.5%	25.3%	29.5%	31.5%	32.1%	32.3%	32.3%	32.3%
4.00	9.1%	20.9%	27.5%	30.9%	32.1%	32.3%	32.3%	32.3%	32.3%

Assumptions: S = X, q = 0%

T = Time to expiration of put option (in years)—i.e., holding period

σ = Annualized volatility of the underlying security

V_0 = Value of the otherwise identical unrestricted interest

The results of the Ghaidarov model closely match the modified Finnerty model for the six-month holding period at volatilities through 125 percent. Unlike the Finnerty model, however, the Ghaidarov model is not limited to the 32.3 percent DLOM threshold. At 150 percent volatility, the DLOM indicated by the model is approximately 25 percent and approaches a DLOM of 100 percent as the volatility increases into perpetuity.

Exhibit 10 presents the Ghaidarov model for various holding periods and volatilities.

Stockdale affirms that the Ghaidarov model behaves properly over time, though cautions against using the model at higher volatilities for long holding periods because the indicated DLOMs with those inputs eventually exceed the discounts implied by the restricted stock studies.[34]

However, with a six-month holding period, the Ghaidarov model indicated DLOMs appear to better conform to the results of the restricted stock studies than the Finnerty model or other POPMs, even at higher volatilities. This point is further illustrated in the summary discussion below.

Long-Term Equity Anticipation Securities

Related to POPMs, Trout published a study analyzing long-term equity anticipation securities (LEAPS) and the DLOM.[35]

Seaman updated the Trout LEAPS study several times—the most recent update was in September 2013.[36]

Each of these LEAPS studies was conducted using a similar research logic and research design.

A LEAPS is essentially a long-term stock option that offers price protection for up to two years into the future. Therefore, an investor who desires protection against stock price declines can purchase a LEAPS put option. The LEAPS studies examined the cost of buying LEAPS put options and concluded that the cost of the LEAPS put option divided by the stock price indicates the DLOM.

Trout examined nine LEAPS as of March 2003 with options expiring January 2005. The nine LEAPS were for large companies with actively traded securities.[37]

According to Trout, "[t]he data concerning the relative cost of puts as an insurance premium indicate an insurance premium cost equal to about 24 percent of the price. This finding suggests that the minimum discount that one should assign for the lack of marketability of holding privately held stock is at least 24 percent."[38]

The Seaman study updated and extended the Trout study through November 2012. The Seaman study examined the relationship between the price of the

Exhibit 10
Indicated DLOM Based on the Ghaidarov Put Option Model
Six-Month Holding Period Highlighted

Holding Period (T, Years)	Volatility (σ)								
	20%	50%	75%	100%	125%	150%	200%	250%	300%
0.25	2.3%	5.8%	8.7%	11.6%	14.6%	17.5%	23.7%	30.0%	36.5%
0.50	**3.3%**	**8.2%**	**12.3%**	**16.5%**	**20.8%**	**25.2%**	**34.2%**	**43.7%**	**53.4%**
0.75	4.0%	10.0%	15.1%	20.4%	25.7%	31.2%	42.7%	54.5%	65.8%
1.00	4.6%	11.6%	17.5%	23.7%	30.0%	36.5%	50.1%	63.4%	75.0%
2.00	6.5%	16.5%	25.2%	34.2%	43.7%	53.4%	71.3%	84.6%	92.8%
3.00	8.0%	20.4%	31.2%	42.7%	54.5%	65.8%	83.5%	93.5%	97.8%
4.00	9.3%	23.7%	36.5%	50.1%	63.4%	75.0%	90.5%	97.2%	99.3%

Assumptions: S = X, q = 0%

LEAPS (i.e., the price discount) and the following variables: (1) company size, (2) company risk, (3) latest year profit margins, (4) latest year return on equity, and (5) company industry.

The Seaman study concluded the following:

- Company size: Revenue size has a major effect on the cost of price protection with smaller levels of revenue associated with larger discounts.
- Company risk: Company risk has a large effect on discounts, with higher risk companies, as measured by a company's beta, associated with a larger discount.
- Latest year profit margin: Company profitability has a mild (but not a major) effect on marketability discounts.
- Return on equity: The company's latest year return on equity has some effect on discounts particularly at the lower end of returns. For positive returns on equity, there is a minor effect on discounts.
- Industry: The size of the discount varies by industry, but the discounts vary even more by the individual company.[39]

Seaman made the following observation with regard to the cost of price protection:

> [T]he costs of price protection are not constant but vary significantly over time. Economic conditions in November 2008 (recession) caused discounts to double or more over the August 2006 period. By November 2009 economic conditions had moderated. The costs of price protection had gone down by about one-third but were still from 30% to 50% above August 2006 levels.[40]

The LEAPS studies concluded that the observed DLOMs are appropriately viewed as benchmark minimum price discounts when applied to privately held companies, for the following reasons:

1. The underlying securities on which the LEAPS were based are often much larger than the privately held subject company.
2. The underlying securities on which the LEAPS were based are marketable.
3. The LEAPS themselves can be sold at any time during the holding period.
4. There is a known liquidity event (i.e., the sale of the underlying security) for LEAPS.

Summary of Put Option Pricing Models

The POPM studies discussed above generally indicate similar price discounts to the aforementioned restricted stock studies given certain volatility assumptions. In the Chaffe, Longstaff (transformed), Finnerty (modified), and Ghaidarov models, the indicated DLOM for a privately held company (assuming volatility between 50 percent and 125 percent) ranges from 8.2 percent to 48.0 percent for a six-month holding period.

In the LEAPS studies, the price discount is much lower, but the authors conclude that the indicated price discount represents a minimum DLOM.

Because of their nature, POPM studies generally only consider the factors that affect option pricing: holding period and volatility. Although other factors are considered in the POPMs, the holding period and volatility factors have the greatest impact on the option prices.

Therefore, POPMs may understate the DLOM, as POPMs ignore other factors that may reduce the marketability for privately held securities (e.g., contractual transferability restrictions). The DLOM indicated by a POPM is an appropriate starting point for a DLOM analysis.

In determining a final DLOM, a valuation analyst should consider other relevant factors that may contribute to a lack of marketability for the subject privately held company interest.

Basing the size of the DLOM initially on the volatility and holding period factors appears reasonable. The holding period relates to the duration of time restricted stock must be held and risk relates to volatility. As the restricted stock studies generally indicate, the longer the required holding period, the greater the price discount a buyer demands.

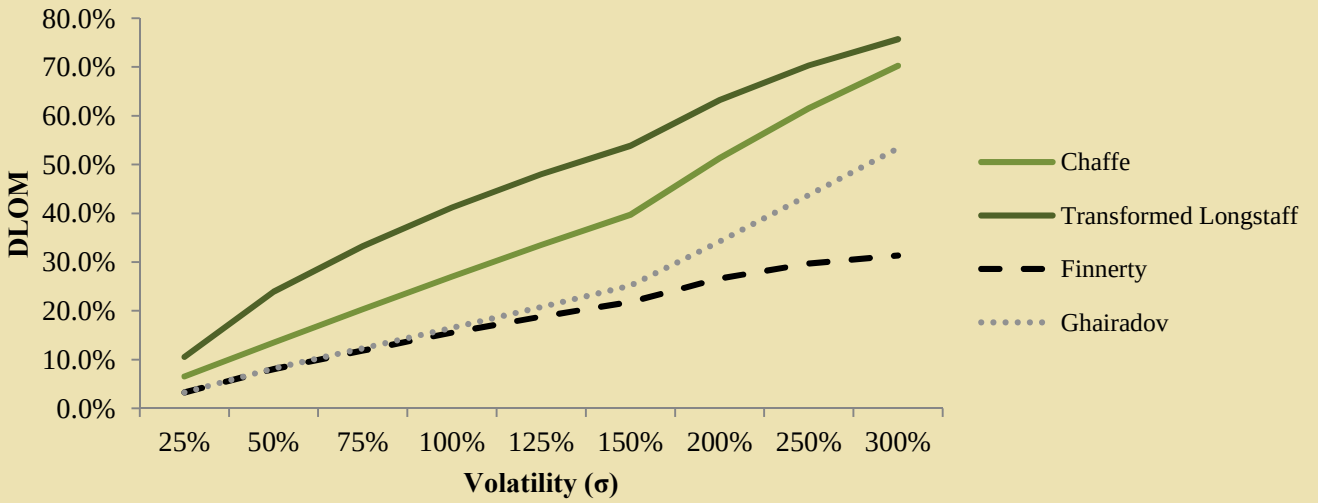
Volatility directly influences the DLOM. When an investor owns a security that is restricted from trading, that investor assumes the risks of:

1. not being able to sell the investment if the value begins to decline and
2. not being able to sell the investment to reallocate funds to another investment.

The first risk factor is affected by highly volatile stocks. Generally, as volatility increases, the risk of stock price depreciation increases along with increases in other risks related to holding a nonmarketable security.

The indicated DLOMs of the POPMs at varying levels of volatility are presented in Figure 2. Some

Figure 2
Put Option Pricing Models
DLOM per Level of Volatility
Based on Assumed Six-Month Holding Period



POPMs are suitable at certain levels of volatility and produce results that appear reasonable. However, no POPM appears to line up closely with the restricted stock transactions at all levels of volatility.

For the purpose of estimating the DLOM for a privately held company interest, the typical range of volatility is 50 percent to 125 percent. Within this range of volatility, the Chaffe, Finnerty (modified), and Ghaidarov models produce reasonable DLOMs that do not exceed the median discounts implied by the restricted stock studies.

The Longstaff model (both the original and transformed) produces results that significantly exceed the discounts implied by the restricted stock studies at volatilities above 50 percent. For this reason, among others, the Longstaff model is seldom applied by valuation analysts without further adjustment to determine the DLOM.

At volatilities above 150 percent, the Finnerty model increasingly produces results that understate the DLOM, as the model produces no discount in excess of 32.3 percent. The Chaffe model suffers the opposite effect—its results spike upward like that of the Longstaff model, eventually exceeding the discounts implied by the restricted stock studies.

As indicated graphically by the results presented in Figure 2, the Ghairadov model is the one that best produces DLOMs that reasonably comport with the discounts implied by the restricted stock studies for all volatilities in the 25 percent to 300 percent range.

As discussed above, the Ghaidarov model and other POPMs provide a useful starting point in determining the DLOM. A valuation analyst should consider the relevance of other factors in addition to the volatility, holding period, and other inputs to the POPM equations.

In *Mandelbaum v. Commissioner*,[41] the Tax Court listed nine factors to consider when determining the DLOM:

1. Financial statement analysis
2. Dividend policy
3. Nature of the company, its history, its position in the industry, and its economic outlook
4. Management
5. Amount of control in the transferred shares
6. Restrictions on transferability
7. Holding period of the stock
8. Company redemption policy
9. Costs associated with a public offering

Further, POPMs were designed to produce results that comport with the discounts of restricted stock studies. But restricted stocks are merely a proxy for estimating the DLOM based on temporary trading restrictions; they do not reflect all of the marketability issues faced by typical privately held companies.

As evidence of this effect, Pluris Valuation Advisors (Pluris) identified two weaknesses with prior restricted stock studies through its LiquiStat study:

1. The lack of measurable parameters with regard to the price discount (for example, was the observed price discount the result of company size or information asymmetry between the buyer and the seller?)
2. The impossibility of establishing two distinct data sets, one completely liquid and one completely illiquid.[42]

Pluris reasoned that the observed price discounts from previous restricted stock studies were likely affected by factors unrelated to illiquidity. These factors including the following:

1. Compensation for control and monitoring
2. Capital scarcity effects
3. Information asymmetry effects

To overcome these perceived weaknesses, Pluris analyzed the pricing of restricted stock in investor-to-investor trades—that is, transactions:

1. not involving the issuer or an affiliate of the issuer and
2. not raising new capital for the issuer.

According to Pluris:

> Clearly, the private placement process has facets, beyond just illiquidity, that affect discounts. The solution, or at least part of the solution, might be to take a look at the pricing of restricted stock in investor-to-investor trades, not involving the issuer or an affiliate of the issuer and not raising capital for the issuer.[43]

The data analyzed was from the LiquiStat database of private sales transactions created by Pluris which it observed in a secondary market established under the SEC Rule 144A section 4(1-1/2) exception.

The transactions in this database appear to provide a direct indication of the fair market value of restricted stock because the buyers and sellers are unrelated to the company and more closely resemble the hypothetical buyers contemplated in the fair market value definition.

The range of indicated DLOM appears to be on the high side compared to past restricted stock studies. Notably, these relatively high DLOMs are derived from transactions with a median holding period of 115 days, or 0.32 years.

In other words, the DLOMs exhibited by the LiquiStat study are associated with a holding period that is much shorter than that which is normally assumed to be present in the restricted stock studies.

As of the date of the Pluris study, the database contained transactions facilitated by Restricted Stock Partners through its Restricted Stock Trading Network.

"The relatively higher implied DLOM of the LiquiStat study . . . suggests that the impact on DLOM of marketability factors other than stock price volatility and holding time can be significant."

There were 61 transactions analyzed in the LiquiStat study. The 61 transactions analyzed in the LiquiStat study were completed at an average price discount of 32.8 percent, and a median price discount of 34.6 percent.

The average number of days remaining before the shares sold became available to trade in the public markets was 144 days, which is comparable to the holding period contemplated under the Section 2704 proposed regulations.

The relatively higher implied DLOM of the LiquiStat study (e.g., 34.6 percent versus 18.7 percent for the FMV study with 75 percent volatility) suggests that the impact on DLOM of marketability factors other than stock price volatility and holding time can be significant.

Conclusion

The Service has issued long-anticipated proposed regulations under Section 2704 that substantially reduce the application of valuation discounts to intrafamily transfers of interests in privately held entities. These regulations may restrict the DLOM to the value of a six-month put option.

As a starting point for a DLOM analysis, there are a number of POPMs that a valuation analyst can apply to determining the appropriate DLOM for a valuation governed by Section 2704.

The Chaffe, Longstaff (transformed), Finnerty (modified), and Ghaidarov models provide an indicated DLOM for a privately held entity in the range of from 8.2 percent to 48.0 percent for a six-month holding period.

These results are based on a typical price volatility for a privately held entity ownership interest in the range of 50 percent to 125 percent.

For price volatilities in the range of 25 percent to 300 percent, the Ghaidarov model is the one that best produces DLOMs that reasonably comport with the discounts implied by the FMV study and other restricted stock studies based on a comparison of its results with that of the restricted stock studies.

The Finnerty model, on which the Ghaidarov model is based, is a generally accepted model for estimating the DLOM, but it tends to understate the DLOM at higher volatilities due to a mathematical limit of 32.3 percent imposed by the equation regardless of increasing volatility and holding period time.

The Chaffe and Longstaff models also are flawed by their application at higher volatilities, resulting in DLOMs that exceed the discounts implied by the restricted stock studies.

The Ghaidarov model and other POPMs provide a useful starting point in determining the DLOM. In determining a final DLOM, a valuation analyst should consider the relevance of other marketability factors in addition to the volatility, holding period, and other inputs to the POPM equations.

Notes:

1. 81 Fed. Reg. 51413-51425 (Aug. 4, 2016).
2. Codified at 26 U.S.C. §2704.
3. Internal Revenue Service, "RIN 1545-BB71: Notice of proposed rulemaking and notice of public hearing," *Federal Register* (August 4, 2016): 5.
4. Ron Aucutt, "Proposed 2704 Regulations Are Significant but Not a Disaster," *Leimberg Estate Pl. Newsletter #2456* (September 28, 2016).
5. Steve R. Akers, "Section 2704 Proposed Regulations," Bessemer Trust white paper (October 2016), available at www.bessemertrust.com/advisor.
6. John D. Finnerty, "Using Put Option-Based DLOM Models to Estimate Discounts for Lack of Marketability," *Business Valuation Review* 31, no. 4 (Winter 2013): 166.
7. Ibid.
8. David B.H. Chaffe III, "Option Pricing as a Proxy for Discount for Lack of Marketability in Privately Held Company Valuations," *Business Valuation Review* (December 1993): 182–6.
9. SEC Rule 144 governs the purchase and sale of stock issued in unregistered private placements. According to the SEC, "When you acquire restricted securities or hold control securities, you must find an exemption from the SEC's registration requirements to sell them in the marketplace. Rule 144 allows public resale of restricted and control securities if a number of conditions are met." See SEC website: http://www.sec.gov/investor/pubs/rule144.htm.
10. Based on John Stockdale Sr., *BVR's Guide to Discounts for Lack of Marketability*, Vol. 1 (2013), 46–54.
11. See, for example, Bruce Johnson, "Restricted Stock Discounts, 1991–95," *Shannon Pratt's Business Valuation Update* (March 1999); Rod Burkert, "Cure for Declining Discounts, Deconstruct the Studies," *Trusts & Estates* (March 2004); and Robert Reilly, "Willamette Management Associates' Discount for Lack of Marketability Study for Marital Dissolution Valuations," *American Journal of Family Law* (Spring 2005).
12. 17 CFR Parts 230 and 239, December 17, 2007.
13. Stockdale, *BVR's Guide to Discounts for Lack of Marketability*, 51–56.
14. The FMV Restricted Stock Study database is available from BVR on a subscription basis. See https://www.bvresources.com/products/the-fmv-restricted-stock-study. Data is updated quarterly.
15. "Determining Discounts for Lack of Marketability: A Companion Guide to the FMV Restricted Stock Study," *Business Valuation Resources* (2016), available at www.bvresources.com.
16. Chaffe, "Option Pricing as a Proxy for Discount for Lack of Marketability in Privately Held Company Valuations": 182–186.
17. Stockdale, *BVR's Guide to Discounts for Lack of Marketability*, 196 and 289.
18. Shannon P. Pratt and Roger J. Grabowski, *Estimating the Cost of Capital—Applications and Examples*, 5th ed. (New York: John Wiley & Sons, 2014), 624–626.
19. Marc S. Katsanis, "Stand Up and Shout—It Is Another DLOM Put Model!" *Business Valuation Review* 31, no. 1 (Spring 2012): 48–52.
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